Does Bill James's Pythagorean Formula Apply to Football?

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ABSTRACT

In 1980, baseball writer and statistician Bill James developed a formula that related a baseball team's win-loss percentage to the number of runs they scored and allowed. This formula was modified in the mid-1990s for use in professional basketball, with points scored and allowed in lieu of baseball's runs scored and allowed. This paper empirically tests the modified formula using data on points for and points allowed for all 32 teams in the National Football League (NFL), by conference, from 2002 through 2020 (and during the two shorter periods, 2002-10 and 2011-20). The authors find that the Bill James method works remarkably well for the NFL. The authors' regressions are also used to identify "overachievers" and "underachievers" during this period.

Introduction

In 1980, Bill James, baseball writer and statistician, developed a formula that related a team's win-loss percentage to the number of runs they scored and allowed, as follows:

$$\text{Win-Loss Percentage} = \frac{(RunsScored)^2}{(RunsScored)^2 + (RunsAllowed)^2} \quad (1)$$

Cha, Glatt, and Sommers [1] empirically tested Bill James's Pythagorean formula using Major League Baseball data on all teams in both leagues from 1950 to 2007. The authors found that Bill James's Pythagorean formula has, in general, stood the test of time and, in particular, that the exponent on "Runs Scored" and "Runs Allowed" has been close to "2."

In 1994, while a researcher at STATS, Inc., Daryl Morey, currently president of the Philadelphia 76ers of the National Basketball Association (NBA), modified Bill James's Pythagorean formula for use in professional basketball. In lieu of baseball's "Runs Scored" and "Runs Allowed," Morey used basketball's "Points Scored" and "Points Allowed" [2]. Morey estimated the exponents on each variable to be "13.91." Hence, the win-loss percentage was given by:

$$\text{Win-Loss Percentage} = \frac{(PointsScored)^{13.91}}{(PointsScored)^{13.91} + (PointsAllowed)^{13.91}} \quad (2)$$

In 2015, using NBA data on all teams, by conference, from 2011-12 through 2012-14, Jackson *et al.* [3] found that Daryl Morey's modified Pythagorean method for all teams in the NBA still explained the NBA's win-loss percentage very well.

In 2009, Cochran and Blackstock [4], using goals scored and allowed, applied Bill James's Pythagorean formula to teams in the National Hockey League. They estimated the exponent on goals scored and allowed to be about 1.93.

Using performance data for the top 100 male singles players between 2004 and 2014, Kovalchik [5] derives a Pythagorean model for match wins in tennis based on the number of break points won.

Initial attempts to apply the Pythagorean formula to soccer using the sport's traditional point system (3 points for a win, 1 for a draw, and no points for a loss) were less successful. (See, for examples, Bertin [6] and Hamilton [7].) One complicating factor is that there are ties (or draws) in soccer, but not in baseball, basketball, hockey (since the NHL instituted a shootout), or tennis. In 2018, Reinmuth and Sommers [8] applied Bill James's Pythagorean method to soccer and, in particular, England's Premier League (EPL). The authors used data on all twenty EPL teams over 17 seasons (2000-01 through 2016-17). When ties (or draws) were excluded, the authors found a stable soccer Pythagorean formula for the EPL, in general, and, in particular, they found that the exponent on goals scored and allowed was about 1.70. Moreover, the ratio of goals scored to goals allowed explained more than 90 percent of the variation in a soccer team's win-loss ratio.

In this paper, the authors were curious to know if Bill James's Pythagorean formula applies as well to American football as it does to European football. Does the method work better for one sixteen-team conference (American Football Conference or National Football Conference) than for the other? Does the method work better over the last ten years in the NFL than over the previous nine?

The Data

In 2002, the National Football League expanded to 32 teams (with the addition of the Houston Texans). The 32-team league was realigned into two conferences, the American Football Conference (AFC) and the National Football Conference (NFC), each with four four-team divisions. The Seattle Seahawks moved from the AFC to the NFC. Otherwise, most divisions remained the same compared to the year before realignment.

Data on regular season wins, losses, points for (that is, points scored by the team over the regular season), and points allowed (points scored by the opposition) for all 32 NFL teams are from www.football-reference.com [9] for all seasons between 2002 and 2020. Over these nineteen seasons, there are 608 observations. Unlike baseball and basketball, in football there are ties. Ties are counted as half-wins and half-losses. For example, a team that wins 10 games, loses 5, and ties once would have a win-loss percentage of .656 [= (10×1 + 1×0.5)/16]. Hereafter, the number of "wins" will be defined as the number of "wins" plus one-half the number of "ties." Similarly, the number of "losses" will be defined as the number of "losses" plus one-half the number of "ties." Over the 19-year period, only 22 NFL games ended in a tie, seven in the AFC and 15 in the NFC. And, unlike baseball with each team playing 162 games in the regular season and basketball with each team playing 82 games, the football season is much shorter with each team playing only 16 games in a regular season (for the period 2002 – 2020).

Methodology

Since the win-loss percentage is the ratio of games won to the total number of games played (games won plus games lost) equation (2) can be re-written as follows:

$$\frac{Wins}{Losses} = \frac{(PointsScored)^{13.91}}{(PointsAllowed)^{13.91}} = \left(\frac{PointsScored}{PointsAllowed}\right)^{13.91} \tag{3}$$

And, equation (3), in turn, can be written in log-linear form as:

$$\ln\left(\frac{Wins}{Losses}\right) = \ln\left(\frac{PS}{PA}\right)^{13.91} \quad (4)$$

where "ln" is the natural logarithm, *PS* denotes points scored, and *PA* denotes points allowed. That is, if one first takes the natural logarithm of both sides of equation (3) and then if we define $y_{i,t} = \ln\left(\frac{Wins}{Losses}\right)_{i,t}$ and $x_{i,t} = \ln\left(\frac{PS}{PA}\right)_{i,t}$ for each team *i* in year *t*, we can estimate the coefficients β_0 and β_1 by applying least squares to *y* and *x* in the following regression:

$$y_{i,t} = \beta_0 + \beta_1 x_{i,t} + \varepsilon_{i,t} \quad (5)$$

where $\varepsilon_{i,t}$ is a disturbance term. According to Bill James, β_0 should be equal to zero and β_1 should be close to 2 for baseball. According to Daryl Morey, β_0 should again be indistinguishable from zero and β_1 should be close to "13.91" for basketball. In the case of football, β_0 should be close to zero and β_1 needs to be estimated. Hereafter, equation (5), where $y_{i,t} = \ln\left(\frac{Wins}{Losses}\right)_{i,t}$ and $x_{i,t} = \ln\left(\frac{PS}{PA}\right)_{i,t}$ will be called Model (1). In estimating equation (5), three teams — the undefeated New England Patriots in 2007 and the winless Detroit Lions in 2008 and Cleveland Browns in 2017 — were excluded since the natural logarithm is defined only for $^{Wins}/_{Losses}$ greater than zero.

Model (1) assumes that one more point scored has the same impact on a team's winning percentage as does one less point allowed. But what if scoring points was more (or less) important to winning games than allowing points? Model (1) might then be revised as follows:

$$\ln\left(\frac{Wins}{Losses}\right)_{i,t} = \beta_0 + \beta_1 \ln(PS)_{i,t} + \beta_2 \ln(PA)_{i,t} + \varepsilon_{i,t} \quad (6)$$

If we relax the assumption that the exponent on the ratio $\left(\frac{PS}{PA}\right)$ is the same (and, according to James equal to "2" or Morey equal to "13.91"), then the revised model would be described by equation (6), hereafter Model (2). To test the null hypothesis H_0: $\beta_1 = \beta_2$, we employ a *t*-test. If we cannot reject H_0: $\beta_1 = \beta_2$, then we will employ Model (1) to compare a team's actual and predicted winning percentage.

The Results

Table 1 shows the regression results for each 16-team conference (as well as for both conferences combined) for each of two periods (2002-10 and 2011-20) and the two periods combined. The estimated intercept (b_0) in *all* regressions is not discernible from zero, as Bill James/Daryl Morey would expect. In all nineteen years, the exponent on the ratio of points scored to points allowed is 2.78. Moreover, the estimate for β_1 is stable despite the fact that significantly more points were scored per team in each conference during regular seasons since 2011 than in the period before 2011 ($\overline{PS}_{AFC,2002-10} = 346.53$, $\overline{PS}_{AFC,2011-20} = 357.72$, two-sided *p*-value on difference is .1835; $\overline{PS}_{NFC,2002-10} = 337.86$, $\overline{PS}_{NFC,2011-20} = 375.67$, two-sided *p*-value on difference is <.0001). No coefficient of determination (R^2) for any of the twelve regressions drops below .796. That is, the correlation between $\ln\left(\frac{Wins}{Losses}\right)$ and $\ln\left(\frac{PS}{PA}\right)$ is better than .89 (that is, the square root of .796) for each conference (or both conferences combined) in each of the two periods (or all nineteen seasons combined).

Table 2 shows the regression results for Model (2), which isolates the impact of points scored from the impact of points allowed on the win-loss ratio. The right-hand column reports the R^2 for each regression. A look down this column and the corresponding column in Table 1 clearly shows that the explanatory power of Model (2) is not an improvement over Model (1). Moreover, the second-to-last column of Table 2 (with two-sided *p*-values on H_0: $\beta_1 = \beta_2$) shows that the coefficient on *PS* is equal (in absolute value) to the coefficient on *PA*. That is, in no instance can we reject H_0: $\beta_1 = \beta_2$ in favor of H_A: $\beta_1 \neq \beta_2$ (using $\alpha = .05$ or even $\alpha = .10$).

Finally, Table 3 uses information on a specific team's points scored to points allowed in a given season (namely, the regressions for 2002-20 and "both conferences" in Table 1) to predict that team's winning percentage. Those teams whose actual winning percentage exceeded their predicted winning percentage by the biggest margin are dubbed "overachievers." Similarly, those teams whose predicted winning percentage exceeded their actual winning percentage by the biggest margin are dubbed "underachievers." Over the entire period 2002-20, the 2020 Atlanta Falcons and the 2012 Indianapolis Colts were, respectively, the biggest underachievers and overachievers. The 2012 Indianapolis Colts advanced to the playoffs, but lost the wild card game (9 – 24) to the Baltimore Ravens, that year's eventual winner of the Super Bowl. Surprisingly, the undefeated 2007 New England Patriots are not listed among the top overachievers in the period 2002 to 2010 because their ratio of points scored to points allowed that season gave the team a predicted win-loss percentage of .895. Not surprisingly, the winless 2017 Cleveland Browns are listed among the three worst underachievers in the period 2011-20.

Concluding Remarks

When Bill James's Pythagorean formula (originally developed for baseball) is applied to professional football, the authors find that points scored and points allowed have equal (and opposite) effects on team winning, in both the AFC and NFC conferences (and both conferences combined) in all games played between 2002 and 2020. The exponent on points scored to points allowed is 2.78, marginally higher than the "2" Bill James found for baseball games.

Table 1. Regression Results for Model (1)
$\ln(WINS/LOSSES) = b_0 + b_1 \ln(PS/PA)$

	Intercept b_0	Slope coefficient on $\ln(PS/PA)$ b_1	R^2
2002-10[a]			
AFC ($n = 143$)	.0072 [.0353][a]	2.7586 [.1177]	.796
NFC ($n = 143$)	.0215 [.0283]	2.5309 [.0958]	.832
Both conferences	.0174 [.0226]	2.6445 [.0758]	.811
2011-20			
AFC ($n = 159$)	.0326 [.0331]	2.8379 [.1120]	.804
NFC ($n = 160$)	-.0027 [.0297]	3.0210 [.1139]	.817
Both conferences	.0164 [.0222]	2.9147 [.0796]	.809
2002-20			
AFC ($n = 302$)	.0198 [.0240]	2.7968 [.0807]	.800
NFC ($n = 303$)	.0142 [.0209]	2.7559 [.0751]	.817
Both conferences	.0170 [.0159]	2.7779 [.0552]	.808

[a]Numbers in brackets are standard errors.

Table 2. Regression Results for Model (2)
$\ln(WINS/LOSSES) = b_0 + b_1 \ln(PS) + b_2 \ln(PA)$

	Intercept b_0	Slope coefficient on: ln(*PS*) b_1	ln(*PA*) b_2	*p*-value on $H_0: \beta_1 = \beta_2$	R^2
2002-10					
AFC	.9977 [1.7597][a]	2.6881 [.1721]	-2.8585 [.2131]	.574	.796
NFC	-.4174 [1.4967]	2.5621 [.1432]	-2.4867 [.1788]	.770	.832
Both conferences	.3126 [1.1583]	2.6234 [.1123]	-2.6741 [.1389]	.799	.811
2011-20					
AFC	-2.0619 [1.8504]	2.9847 [.1712]	-2.6281 [.2165]	.259	.805
NFC	1.8356 [1.5187]	2.8842 [.1603]	-3.1955 [.1835]	.228	.818
Both conferences	.2548 [1.1763]	2.8977 [.1158]	-2.9382 [.1405]	.839	.809
2002-20					
AFC	-.5822 [1.2228]	2.8401 [.1194]	-2.7371 [.1458]	.623	.800
NFC	1.0681 [1.0215]	2.6817 [.1040]	-2.8614 [.1268]	.303	.818
Both conferences	.2895 [.7889]	2.7585 [.0788]	-2.8053 [.0959]	.730	.808

[a]Numbers in brackets are standard errors.

Table 3. Overachieving and Underachieving NFL Teams, 2002 through 2020 Seasons

	Biggest Overachievers			Biggest Underachievers	
	Actual winning pct.	Predicted winning pct.		Actual winning pct.	Predicted winning pct.
2002-10					
Pittsburgh Steelers 2004	.938	.752	Jacksonville Jaguars 2006	.500	.702
Atlanta Falcons 2004	.688	.510	Green Bay Packers 2008	.375	.572
Indianapolis Colts 2009	.875	.703	Baltimore Rayens 2009	.563	.758
2011-20					
Indianapolis Colts 2012	.688	.448	Atlanta Falcons 2020	.250	.473
Kansas City Chiefs 2011	.438	.218	Dallas Cowboys 2019	.500	.702
Cleveland Browns 2020	.688	.486	Cleveland Browns 2017	.000	.176
2002-20					
Indianapolis Colts 2012	.688	.448	Atlanta Falcons 2020	.250	.473
Kansas City Chiefs 2011	.438	.218	Jacksonville Jaguars 2006	.500	.702
Cleveland Browns 2020	.688	.486	Dallas Cowboys 2019	.500	.702

References

1. D.U. Cha, D.P. Glatt, and P.M. Sommers, "An Empirical Test of Bill James's Pythagorean Formula," *Journal of Recreational Mathematics*, 35:2, pp. 117-124, 2009.

2. J. Dewan and D. Zminda, *STATS Basketball Scoreboard, 1993-94*, STATS, Inc., p. 17, October 1993.

3. A.S. Jackson, J.R. Piper, J.R. Jensen, and P.M. Sommers, "Does Bill James's Pythagorean Formula Apply to Basketball?" *Topics in Recreational Mathematics* (ed. by Charles Ashbacher), Vol. 1(1), pp. 7-18, 2015.

4. J.J. Cochran and R. Blackstock, "Pythagoras and the National Hockey League,"*Journal of Quantitative Analysis in Sports*, Vol. 5(2), pp. 1-13, 2009.

5. S.A. Kovalchik, "Is There a Pythagorean Theorem for Winning in Tennis?" *Journal of Quantitative Analysis in Sports*, Vol. 12(1), pp. 43-49, 2016.

6. M. Bertin, "Improving Soccer's Version of the Bill James Pythagorean," *StatsBomb*, April 26, 2016, http://statsbomb.com/2016/04/improving-soccers-version-of-the-bill-james-pythagorean/.

7. H. Hamilton, "Why the Baseball Pythagorean Doesn't Work for Soccer," *Soccermetrics Research*, April 21, 2016, http://www.soccermetrics.net/soccerpythagorean-theory/why-the-baseball-pythagorean-doesnt-work-for-soccer.

8. K.A. Reinmuth and P.M. Sommers, "Does Bill James's Pythagorean Formula Apply to England's Premier League?" *Journal of Student Research*, Vol. 7(2), 2018, pp. 42-48.

9. Pro-Football-Reference.com. https://www.pro-football-reference.com/years/2020/;
https://www.pro-football-reference.com/years/2019/index.htm;
https://www.pro-football-reference.com/years/2018/index.htm;
https://www.pro-football-reference.com/years/2017/index.htm;
https://www.pro-football-reference.com/years/2016/index.htm;
https://www.pro-football-reference.com/years/2015/index.htm;
https://www.pro-football-reference.com/years/2014/index.htm;
https://www.pro-football-reference.com/years/2013/index.htm;
https://www.pro-football-reference.com/years/2012/index.htm;
https://www.pro-football-reference.com/years/2011/index.htm;
https://www.pro-football-reference.com/years/2010/index.htm;
https://www.pro-football-reference.com/years/2009/index.htm;
https://www.pro-football-reference.com/years/2008/index.htm;
https://www.pro-football-reference.com/years/2007/index.htm;
https://www.pro-football-reference.com/years/2006/index.htm;
https://www.pro-football-reference.com/years/2005/index.htm;
https://www.pro-football-reference.com/years/2004/index.htm;
https://www.pro-football-reference.com/years/2003/index.htm;
https://www.pro-football-reference.com/years/2002/index.htm .