Robust Control of an Electromagnetic Suspension System: H_2, H_∞ and μ-Synthesis Approach

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ABSTRACT

The electromagnetic suspension system is a motivating and inspiring project for students to investigate the primary principles of electrical engineering, such as control theory, including robust performance and modeling uncertainty. This paper studies robust control methods for an electromagnetic suspension system with H_2, H_∞ and μ-synthesis control approaches to provide robust system performance in the presence of uncertainties and disturbances. System uncertainties are critical parts of the interconnected system modeling, affecting its performance. The control methods tackle the system uncertainties, including unmodeled dynamics, parametric uncertainty, and linearization error. Robust performance analysis for all proposed control methods is provided, and closed-loop system performance is illustrated. The performance robustness is validated using illustrative simulation results.

Introduction

In various industrial purposes to reduce average power requirements, electromagnetic suspension is used to stabilize the levitation, and the static lift against gravity that is provided by a second permanent magnet system, often pulled towards a relatively inexpensive soft ferromagnetic material such as iron or steel [1]. The magnetic suspension system is an alternative automotive suspension system that utilizes magnetically controlled dampers, or shock absorbers, for a highly adaptive smooth ride and manufacturing trains suspending magnetic railways. However, the magnetic suspension system is highly non-linear, including various uncertainties, and its active-controlled type is inherently unstable. Hence, changing the system operating points regarding the model uncertainties is a challenge to construct a high-performance feedback control of magnetic suspension systems [2].

Various studies propose different control methods to achieve high-performance feedback control of magnetic suspension systems and address the aforementioned challenges. Robust control methodologies can be taken to cope with magnetic suspension system uncertainties to guarantee system robust performance in the presence of disturbance compared to the conventional proportional-integral-derivative (PID) controller employed as a feedback compensator. [1] and [3] propose a μ-synthesis approach to control a magnetic suspension system with modeling of uncertainty structure of the system. [4] presents a decomposed fuzzy PID controller with better performance over a typical operational range than a traditional linear PID controller. Fuzzy set theory has evolved as a powerful modeling tool that can cope with the uncertainties and nonlinearity of modern control systems [5]. A pulse-width-modulated (PWM) control method for a magnetic suspension system is studied in [6]. Moreover, the dynamic behavior of the linearized model is studied for control-loop analysis and for compensation in such an unstable system in different operating points. A linear matrix inequality (LMI) based approach is suggested for the position tracking problem of a magnetic levitation system in the presence of parametric uncertainties in [2]. [7] studies a discontinuous integral controller implemented to a magnetic suspension system to provide robust tracking of a time-varying reference in the presence of time-varying perturbations or uncertainties. In [8] an optimal H_∞ control method is suggested for magnetic suspension system where generalized H_∞ norm of the linearized system is used as the optimality criterion. Also, an improved double power reaching law integral sliding mode control (SMC) algorithm to overcome chattering, large overshoot, and slow response of magnetic suspension system is expressed by [9]. However, these studies show that guaranteeing a robust performance of such an uncertain nonlinear system is still challenging, in particular, in the presence of disturbances. To overcome the aforementioned challenges, the objective of the investigated methods in this study is to provide stability and robust performance of an electromagnetic suspension system by modeling all relevant uncertainties, including dynamic uncertainties, parametric uncertainties, and linearization error [1]. More specifically, this paper studies robust control techniques, including H_2, H_∞ and μ-synthesis of electromagnetic suspension system with comparing the robustness and system performance implementing the proposed techniques in the presence of uncertainties and disturbances. Comprehensive simulations are described to validate the theoretical principles. The results of this paper can be used as a project in an advanced undergraduate course in modern control or a graduate course in robust control methods.

This paper is organized as follows: Problem formulation presents the electromagnetic suspension system including applicable structured uncertainties. Control design describes the proposed robust control techniques, H_2, H_2 and μ-synthesis, designed based on an uncertain linear model. Simulation results by implementing the controller on the nonlinear model are presented in numerical result to validate the efficiency of the suggested control methods, followed by the conclusion.

Problem Formulation

An electromagnetic suspension needs to be controlled due to the inherent instability and uncertainties in the system. To find the optimal robust control method for the system, an accurate and proper system model is vital. In this section, the developed model definition, including different types of uncertainties, is described [1]. In this framework, the determined structured uncertainties of the system are parametric uncertainties, unmodeled dynamics, and linearization errors. The closed-loop system should be designed to be robust against disturbances and the mentioned uncertainties. The general principal components of the magnetic suspension system is illustrated in Fig.1 [1]. The control problem is to levitate the iron ball stability utilizing the electromagnetic force.

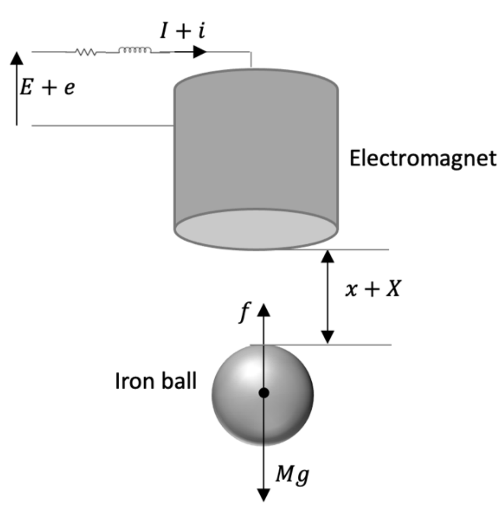

Figure 1. Magnetic Suspension System.

The nonlinear model of the magnetic suspension representing the motion of the iron ball, electromagnetic force, and equation of an electric circuit of the electromagnet can be expressed by (1)-(3) [1].

$$M\frac{d^2x(t)}{dt^2} = Mg - f(t) \quad (1)$$

$$f(t) = k\left(\frac{I+i(t)}{X+x(t)+x_0}\right)^2 \quad (2)$$

$$L\frac{di(t)}{dt} + R\big(I+i(t)\big) = E + e(t) \quad (3)$$

where $f(t)$ is the electromagnetic force which directly impacts $x(t)$, the deviation from the steady-state gap X, and M is the mass of the iron ball. $e(t)$ denotes the deviation from the steady-state voltage E. Here, the steady state current is represented by I and the deviation from steady state current is denoted by i, where x_0 and k are the coefficient of the function $f(t)$. R and L represent the electromagnet resistance and inductance in (3), respectively. The exact description of this nonlinear system should be achieved by infinite-dimensional nonlinear differential equations, where the resulting model is only effective for the simulations or analysis but can not be used for a control system design procedure because of its computational complexity [1]. Linearizing around the operating point yields the nominal transfer function (G_0) defined as:

$$G_0 := \frac{-K_i}{(Ms^2-K_x)(Ls+R)} \quad (4)$$

where $K_x = \frac{2kI}{(X+x_0)^2}$ and $K_i = \frac{2kI^2}{(X+x_0)^3}$ are the linearization error gain [1].
Therefore, the nominal block diagram of the magnetic suspension system is represented in Fig. 2 to use for the control design procedure in this study.

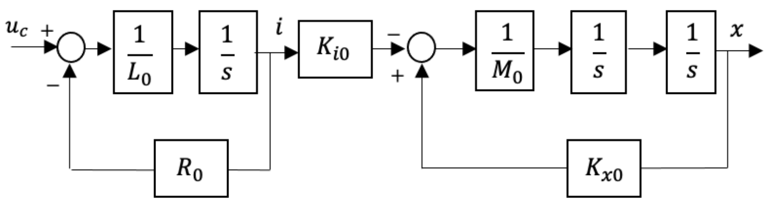

Figure 2. Nominal linear model for the magnetic suspension system.
Modeling Uncertainties of the proposed model can be categorized as follows:

- Linearization error
- Parametric uncertainty
- Unmodeled dynamics

Linearization Error

There are some model uncertainties caused by the linearization of the electromagnetic force about the operating point. The linearization error is represented by sector bounds as:

$$K_i = K_{i0} + k_i\delta_i \qquad \delta_i \in [-1,1] \quad (5)$$

$$K_x = K_{x0} + k_x\delta_x \qquad \delta_x \in [-1,1] \quad (6)$$

where K_{i0} and K_{x0} are nominal values, k_i and k_x are the maximal magnitudes of uncertainties for K_i and K_x, respectively.

Parametric Uncertainty

The mass of the iron ball M is uncertain and is represented by

$$M = M_0 + k_M \delta_M \qquad \delta_M \in [-1,1] \tag{7}$$

where M_0 is the nominal value, and k_M is the uncertainty bound.

Unmodeled Dynamics

The electromagnet has dynamics in the frequency domain given by $\frac{1}{(Ls+R)}$. The inductance L and the resistance R of the electromagnet have frequency-dependent characteristics, and their values are perturbed with frequency change. Moreover, measurements of these parameters are very sensitive. The dynamics of the electromagnet can be represented by an additive uncertainty model as:

$$\frac{1}{Ls+R} = \frac{1}{L_0 s+R_0} + W_{add}(s)\Delta_{add}(s) \tag{8}$$

where L_0 and R_0 are the nominal values of the inductance L and the resistance R, respectively. $W_{add}(s)$ is a known weighting function which represents the uncertainty profile, and $\Delta_{add}(s)$ is unknown but satisfies $\|\Delta(j\omega)\| \leq 1$, for all frequencies ω.

Control Design

The system considered in this section is described by the standard block diagram shown in Fig. (3a) for the H_2 and H_∞control methods. The interconnected systems, including uncertainties, may be rearranged to fit the general framework that is illustrated in Fig. (3b) as a standard μ-Δ configuration for the μ-synthesis control design. In both frameworks, z denotes the controlled signals, d represents the external signals, u_c is the control signal, and y is the measured outputs. In Fig. (3b), Δ represents the set of all possible uncertainties of plant extracted from the system dynamics. The structured uncertainty block (Δ) may include structured unmodeled dynamics and parametric perturbation. In the simplest form, we have either $\Delta = 0$ problem in Fig. (3b) becomes the standard H_∞ control problem shown in Fig. (3a) [10], [11].

This section expresses the state-space dynamic model of the system to use in the control design procedures for robust performance of the electromagnetic suspension system based on dynamic H_2, H_∞ and μ-synthesis control methods.

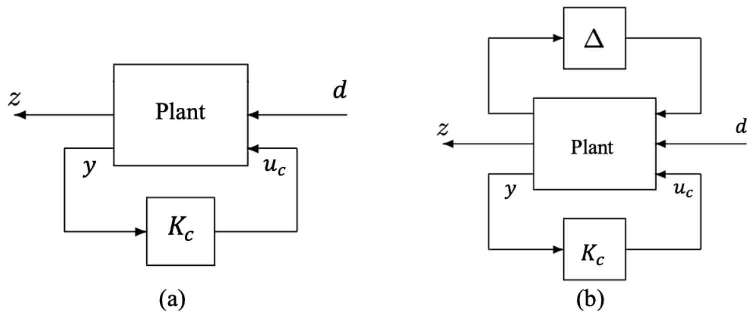

Figure 3. General frameworks. (a) standard block diagram for H_2 and H_∞control methods. (b) standard μ-Δ configuration for the μ-synthesis control design.

State Space Dynamic Model

A state-space dynamic model of the system in Fig. 2 should be provided for design control procedures. Let us consider a stable linear time-invariant system as illustrated by the n dimensional state-space model in (9).

$$\dot{x}(t) = Ax(t) + Bu(t); \quad y(t) = Cx(t) + Du(t) \tag{9}$$

where A, B, C, and D are the state matrices, $u(t)$ is the control input, y(t) is the output of the plant model. Therefore, the state-space representation of the nominal physical plant is defined as:

$$\dot{x}(t) = A_0x(t) + B_0u(t); \quad y(t) = C_0x(t) + D_0u(t) \tag{10}$$

The primary aim of the controller is to guarantee the robust performance of the system in the presence of uncertainties and disturbances. That is controlling the electromagnet to keep the suspended object at the desired position and preserve displacement close to zero. The state-space dynamic model, with specified weighting function, is used to design the robust controllers in the following sections considering the nominal feedback structure in Fig. 4. Weighting functions are mainly used to improve system performance regarding cost function, controlled output, and disturbance effects in the control design procedure. There are various suggestions for choosing the weighting functions that may depend on the designer's skills and involves several iterations until a final form is achieved [12]. However, a theoretical guide for choosing the weighting functions to achieve a functional closed-loop system performance is investigated in [13].

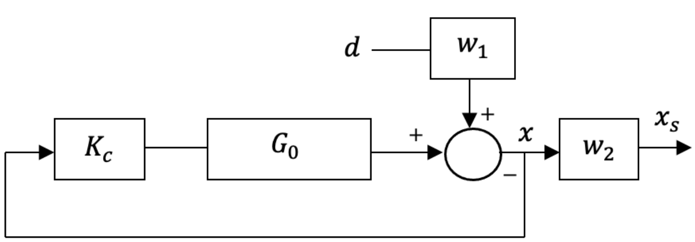

Figure 4. Nominal closed loop structure.

H_2 Control

H_2 control approach is the optimal control of linear time-invariant systems with a quadratic performance criterion [10]. The system considered in this section is described by the general standard block diagram as shown in Fig. (3a).

By considering the dynamical system expressed in (10), the H_2 control problem is to find a proper, real rational controller K_{H_2} that stabilizes G_0 internally and minimizes the H_2 norm of the transfer matrix G_{zd} from d to z [10].

$$\|G_{zd}\| \leq \gamma \quad \text{for some} \quad \gamma > 0 \tag{11}$$

Consider the realization of the plant transfer matrix G expressed by

$$G = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix} \tag{12}$$

where D_{11} and D_{22} are assumed to be zero to make the transfer function G_{22} strictly proper and guarantee that H_2 problem is well defined.
Assume we have an n-dimensional invariant spectral subspace $dom(Ric) \subset \mathbb{R}^{2n\times 2n} \rightarrow \mathbb{R}^{n\times n}$ where Ric denotes a function $H \rightarrow X_n$. Thus, $X_n \in \mathbb{C}^{n\times n}$ can be uniquely determined by Hamiltonian matrices H where it has no eigenvalues on the imaginary axis. Therefore, it has n eigenvalues in $Re\{s\} < 0$, and n in $Re\{s\} < 0$ [10]. Here, the following assumptions are made for the output feedback H_2 problem for stabilizability of G by output feedback, nonsingularity of the H_2 problem, and guaranteeing that the obtained Hamiltonian matrices associated with the following H_2 problem belong to $dom(Ric)$, respectively [10].

- (A, B_2) is stabilizable and (C_2, A) is detectable.
- $R_1 = D_{12}^* D_{12} > 0$ and $R_2 = D_{21} D_{21}^* > 0$.
- $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank for all ω.
- $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank for all ω.

In this study "h2syn" command from MATLAB robust control toolbox is used which returns a controller K_c that stabilizes G. The closed-loop system $CL = F_L(G, K_c)$ achieves the performance level gamma, which is the H_2 norm of CL, where F_L represents the linear fractional transformation (LFT).

H_∞ Control

This section expresses the theoretical formulation of the output H_∞ control problem briefly [10] for the magnetic suspension system under the general framework illustrated in Fig. (3a). The H_∞ control method is an optimization control problem which minimizes the infinity-norm of the lower LFT, $F_L(G, K_c)$, as described in (13).

$$\|F_L(G, K_c)\|_\infty < \gamma' \tag{13}$$

where $F_L(G, K_c)$ denotes the transfer function matrix of the nominal closed-loop system (G_{zd}) from the disturbance signal to the controlled output signals [10] expressed in (14).

$$F_L(G, K_c) := G_{11} + G_{12} K_c (I - G_{22} K_c)^{-1} G_{21} \tag{14}$$

Here, G_{11}, G_{12}, G_{21}, and G_{22} are the partitions of G satisfying

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} d \\ u_c \end{bmatrix} \tag{15}$$

Using the H_∞ control method, the objective is finding a controller K_c such that the obtained closed-loop system is internally stable and

$$\|G_{zd}\|_\infty \leq \gamma'; \quad \text{for some} \quad \gamma' \qquad (16)$$

However, the solution for this optimization problem is not unique \cite{essential} since there is no analytic method to solve it. This study uses "hinfsyn" command in MATLAB to find the dynamic controller K_c. Note that even if the augmented plant is a generalized state-space model with uncertainties or tunable control design blocks, then "hinfsyn" uses the nominal value of all uncertain elements to find the controller K_c. Simulation results by applying this control method are provided in simulation results.

μ-Syntheis Control

There are various software packages, such as μ analysis and synthesis toolbox, that are available to generate the interconnection structure from system components [10]. The general form of this structure, including system uncertainties denoted by Δ, is shown in Fig. (3b). This paper uses the μ -synthesis control method to design a robust controller K_c to stand with the system uncertainties and unknown disturbances. The μ-synthesis problem for the general optimization in (17) is not fully solved yet [10].

$$\min_{K_c} \|F_L(G, K_c)\|_\alpha; \quad \text{for} \quad \alpha = 2 \text{ or } \infty \text{ and } \mu \qquad (17)$$

However, μ may be identified by scaling and applying $\|.\|_\infty$-norm by a reasonable approach known as the D-K iteration method defined as:

$$\min_{K_c} \quad \inf_{D,D^{-1} \in H_\infty} \quad \|DF_L(G, K_c)D^{-1}\|_\infty \qquad (18)$$

where D expresses a positive definite symmetric matrix with appropriate dimension as a minimum phase scaling matrix and satisfies $D(s)\Delta(s) = \Delta(s)D(s)$ [10]. With either K_c or D fixed, the global optimum in the other variable may be found using the μ and H_∞ solutions [15]. The structured uncertainties to design K_c using the μ-synthesis method may include structured unmodeled dynamics, linearization error and parametric perturbations. The D-K iteration process can be defined as follows:

- Using H_∞ synthesis to find a controller that minimizes the closed-loop gain of the nominal system.
- Performing a robustness performance analysis to estimate the robust H_∞ performance of the closed-loop system. This quantity is denoted as a scaled H_∞-norm, including the minimum phase scaling matrix D. (the D step)
- Finding a new controller to minimize the scaled H_∞-norm obtained in the second step. (the K step)
- Returning to the second and the third steps until robust performance stops improving.

The D-K iteration process can be done with "musyn" from the MATLAB toolbox to obtain the optimal dynamic control K_c. The upper bound of μ_u of the robust H_∞ performance for the current controller H_∞ can be obtained in the D step. The D step starts getting the robust performance for the closed-loop uncertain system $F_L(G, K_c)$. The proposed robust control method based on μ-synthesis is defined to overcome the various type of uncertainties in the system. This method can provide robust control for the electromagnetic suspension system in the presence of uncertainties and disturbances. Simulation results to validate the effectiveness of the proposed control method are illustrated in simulation results.

Numerical Results

In this section simulation results are expressed considering the nominal model defined as

$$G_{nominal} = \frac{-30.245}{(s^2-64.5^2)(s+47.5)} \quad (19)$$

Then we can compute the generalized model including the weighting functions w_1 and w_2 using "sysic" command in MATLAB. As it can be seen from Fig. 4, the weighting functions w_1, and w_2 are chosen to improve the system performance in control design procedure where a proper selection of the weighting functions are $w_1 = \frac{100}{(0.01s+1)}$ and $w_2 = \frac{1695}{(1+30s)}$.

$$A_0 = \begin{bmatrix} -47.5 & 4160 & 1.97\times10^5 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -121 & -0.1 \end{bmatrix},\ B_0 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 4 & 0 \end{bmatrix},\ C_0 = \begin{bmatrix} 0 & 0 & 0 & 4.125 \\ 0 & 0 & 30.24 & 0 \end{bmatrix},\ D_0 = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \quad (20)$$

Here, the parameters related to the uncertainties are set as $K_{i0} = 29.64$ N/A, $K_{x0} = 7.28 \times 10^3$ N/m, $k_i = 6.60$, $k_x = 0.57 \times 10^3$, $M_0 = 1.75$ kg, and $k_M = 0.05$. The steady-state gap is set to 5 mm. The nominal transfer function of the unmodeled dynamic uncertainty function can be defined in (21), where W_{add} is given in (22).

$$\frac{1}{L_0 s+R_0} = \frac{1}{(0.56s+26.60)} \quad (21)$$

$$W_{add} = \frac{1.9\times10^{-5}s^2+0.2s+23.8}{s^2+89.97s+7075} \quad (22)$$

Simulation results are provided based on two different cases wherein Case 1, the H_2 and H_∞ controllers are designed with no uncertainty for disturbance rejection purposes. In Case 2, by incorporating the uncertainties in the model as described in problem formulation, a μ-synthesis controller following the D-K iteration procedure using the MATLAB command "musyn" to achieve the robust performance is designed and implemented on the system. All controllers are applied to the nonlinear model to make the resulting performance close to the realistic model. Moreover, the μ-analysis test is performed to check if the designed controller achieves robust stability and performance by measuring the lower-bound and upper-bound of the stability margin. The MATLAB command that can be used is "robuststab".

Case 1

In this scenario, the nominal model $G_0(s)$ is used to design the H_2 and the H_∞ controllers by taking d as the external disturbance and x as the controlled output. The simulation result showing the gap deviation from the steady-state gap (X) is illustrated in Fig. 5. Clearly, using both H_2 and H_∞ controllers can reduce the deviation from the steady gap between the electromagnet and the iron ball, $x(t)$. As it is shown, this deviation becomes zero quickly where the disturbance is considered as an external force equal to 10 N. The performance level γ for the H_2 controller is 1.52 and the observed γ' related to the H_∞ controller is 0.0343. The H_∞ controller outperforms the H_2 controller by providing a minor gap deviation.

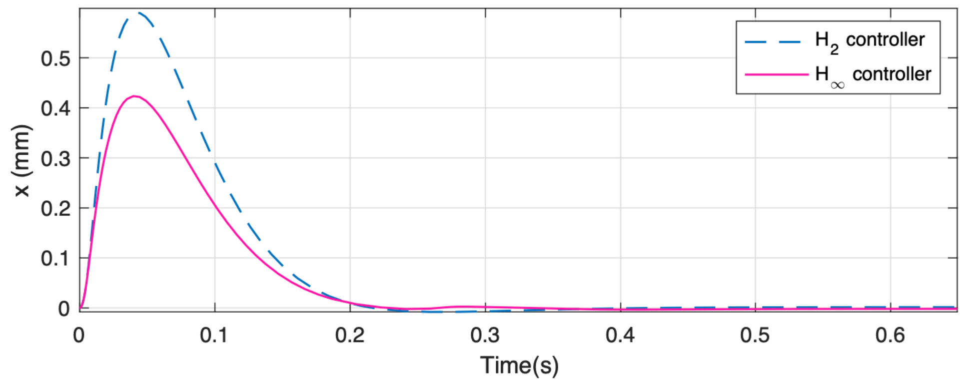

Figure 5.Closed-loop response of nonlinear system in the presence of disturbance ($d = 10$ N).

Case 2

As a second scenario, a μ-synthesis controller is designed for the uncertain system, and its robust performance and stability achievement are analyzed. Here, we need to make our generalized plant by considering all uncertainties for the system. Then "musyn" or "dksyn" MATLAB toolbox can be used to obtain the dynamic, robust controller. The deviation for the desired gap between the iron ball and electromagnet is illustrated in Fig. 6 for different scenarios, including various range of uncertainties for the mass of iron ball, and magnitude of K_i and K_x as a linearization error gain, where each scenario has the following uncertainty range:

- Scenario I: $\delta_x = \delta_i = \delta_M = 0$
- Scenario II: $\delta_x = \delta_i = \delta_M = 1$
- Scenario III: $\delta_x = \delta_i = \delta_M = -1$

In the scenario I, all uncertain parameters are considered their nominal value, wherein scenarios II and III carry their maximum and minimum value, respectively.

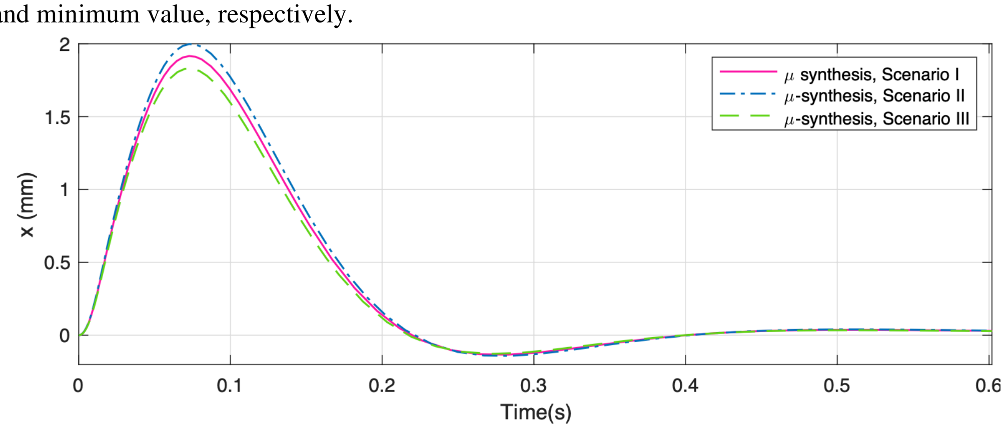

Figure 6. Closed-loop response of nonlinear system in the presence of disturbance ($d = 10$ N) with three different uncertainty ranges.

The lower and upper bounds of $\mu[F_L(G, K_c)]$ are shown in Fig. 7. The bode diagram of the final balanced μ-synthesis controller is also presented in Fig. 8 where the best achieved robust performance is about 0.492. This result means that the gain from disturbance signal d to the controlled output remains below 0.492 for up to $1/0.492$ times the uncertainty specified in the plant. Thus, the controller achieves robust performance objectives for the full range of modeled uncertainties. In addition, we can examine the robust performance of the proposed structure by computing the worst-case gain of the closed-loop system. In this study, the result confirms that the actual worst-case gain over the modeled uncertainty is about 0.2915, which is within the robust performance of 0.492 guaranteed by "musyn".

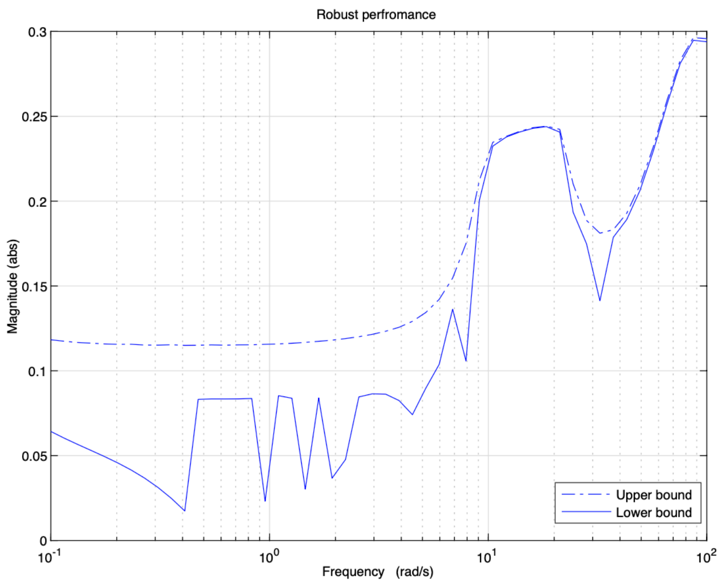

Figure 7. Lower and upper bound for μ-synthesis.

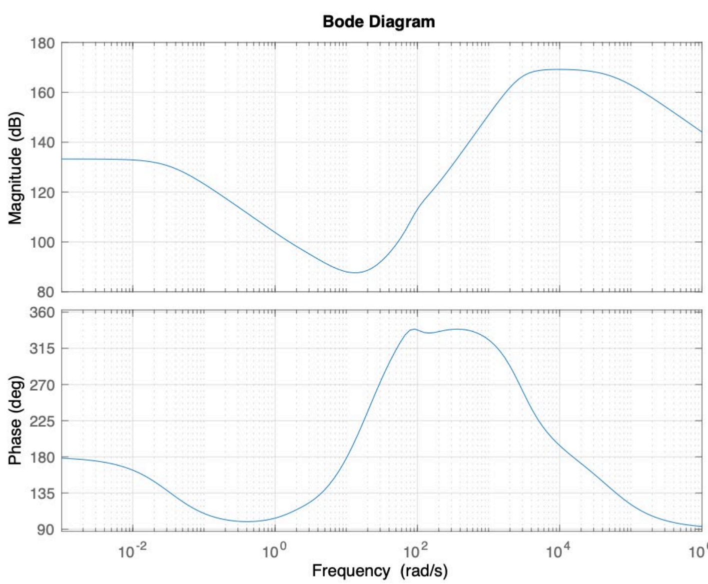

Figure 8. Frequency response of μ-synthesis controller.

Conclusion

This paper investigates the H_2, H_∞ and μ-synthesis control approaches to provide robust performance for an electromagnetic suspension system, including all system uncertainties and possible disturbances. All relevant uncertainties, including dynamic uncertainties, parametric uncertainties, and linearization error, are considered in this study in the control design procedure. Comprehensive results showing the deviation from the desired gap between the iron ball and the electromagnet are provided by applying the dynamic, robust controllers on the nonlinear system in different scenarios. This study, including the control design procedures and modeling uncertainty of the system, can be considered as an inspiring project for students to investigate the primary principles of control engineering in an advanced undergraduate course in modern control or a graduate course in robust controller design.

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